Parks Puzzle is NP-Complete

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The Parks Puzzle Decision Problem

The general *Parks Puzzle* decision problem is to determine, given a board with some marked trees, if there is some configuration of the board that is a valid solution to the puzzle. In our paper we show that the restricted decision problem without any marked trees, Parks, is NP-Complete, proving a lower bound for the general decision problem. If the decision problem were to be solvable in polynomial time, then the function problem of finding an explicit solution to a Parks Puzzle would be trivially solvable in polynomial time. However, our proof that Parks is NP-Complete suggests that it is very unlikely that there exist any efficient algorithms for the decision as well as function problems.

The Rules

A parks puzzle consists of an n x n grid with n contiguous regions known as parks, each marked with a different colour on the grid. Each square may be marked by a tree, represented by T, or an X, which is used to indicate that a square does not contain a tree, or may simply be left empty. A solution to a parks puzzle is a configuration such that,

- Each row, column and park contains one tree
- No two trees are on squares that border one-another

3SAT

We show that Parks is NP-Hard by a reduction from the NP-Complete problem 3SAT. 3SAT is the decision problem of determining whether there exists some assignment of variables that satisfies a given boolean expression in 3CNF form, which have the general form

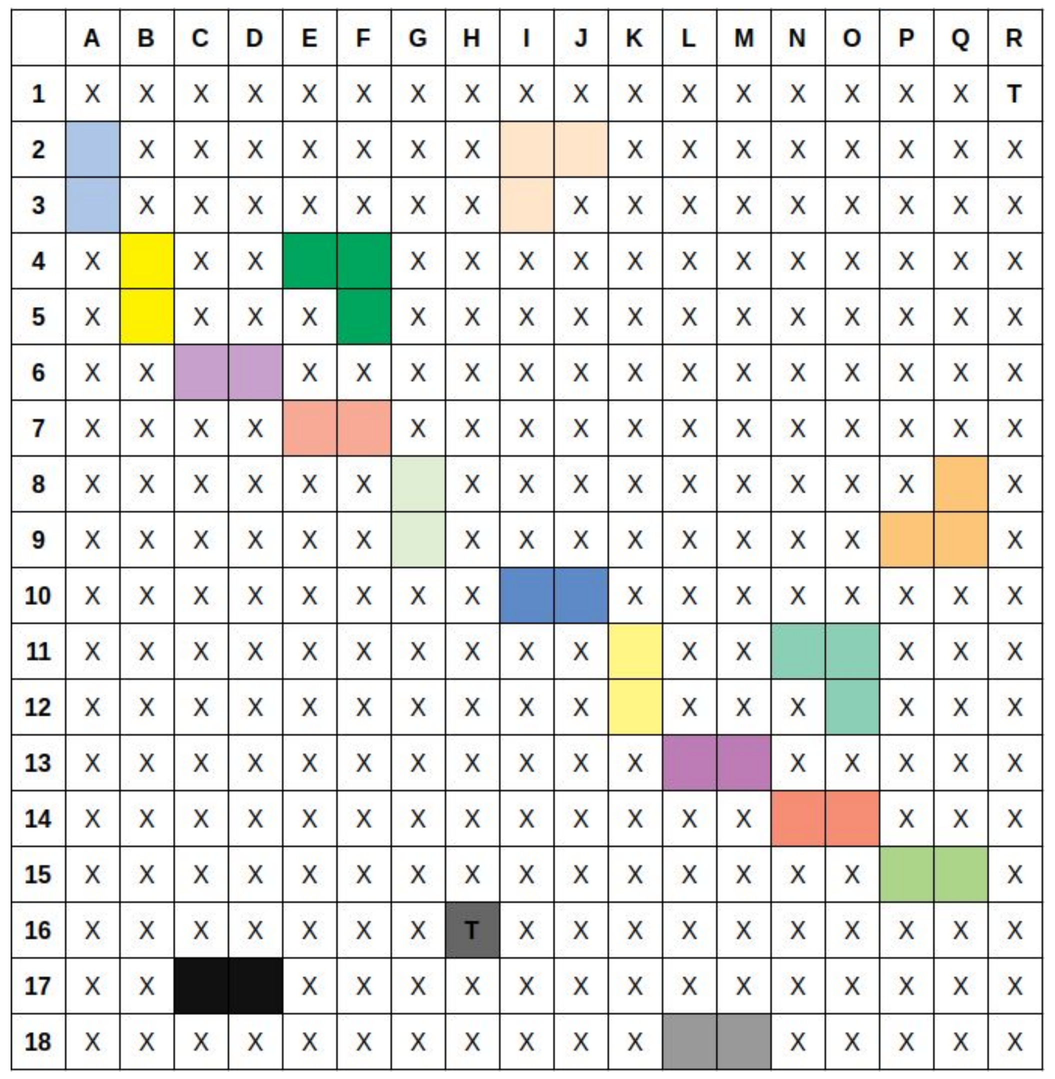
$$(X_1 \vee X_2 \vee X_3) \wedge (X_4 \vee X_5 \vee X_6) \wedge \ldots$$

Proof Sketch

Parks is trivially in NP. We show that Parks is NP-Hard by designing a scheme for representing 3SAT expressions as Parks Puzzle such that an assignment of variables in the puzzle is consistent IFF the same assignment of variables satisfies the boolean expression. Each ternary disjunction is represented by an OR gadget, and the variables are equated by IFF gadgets. All of the gadgets are placed so that they do not interfere with one-another, and any assignment of variables must satisfy every one of these disjunctions for the puzzle to be consistent, which means that there is no need for an AND gadget. Note that a binary park, or variable park is said to be True IFF it's topmost, or leftmost square contains a tree

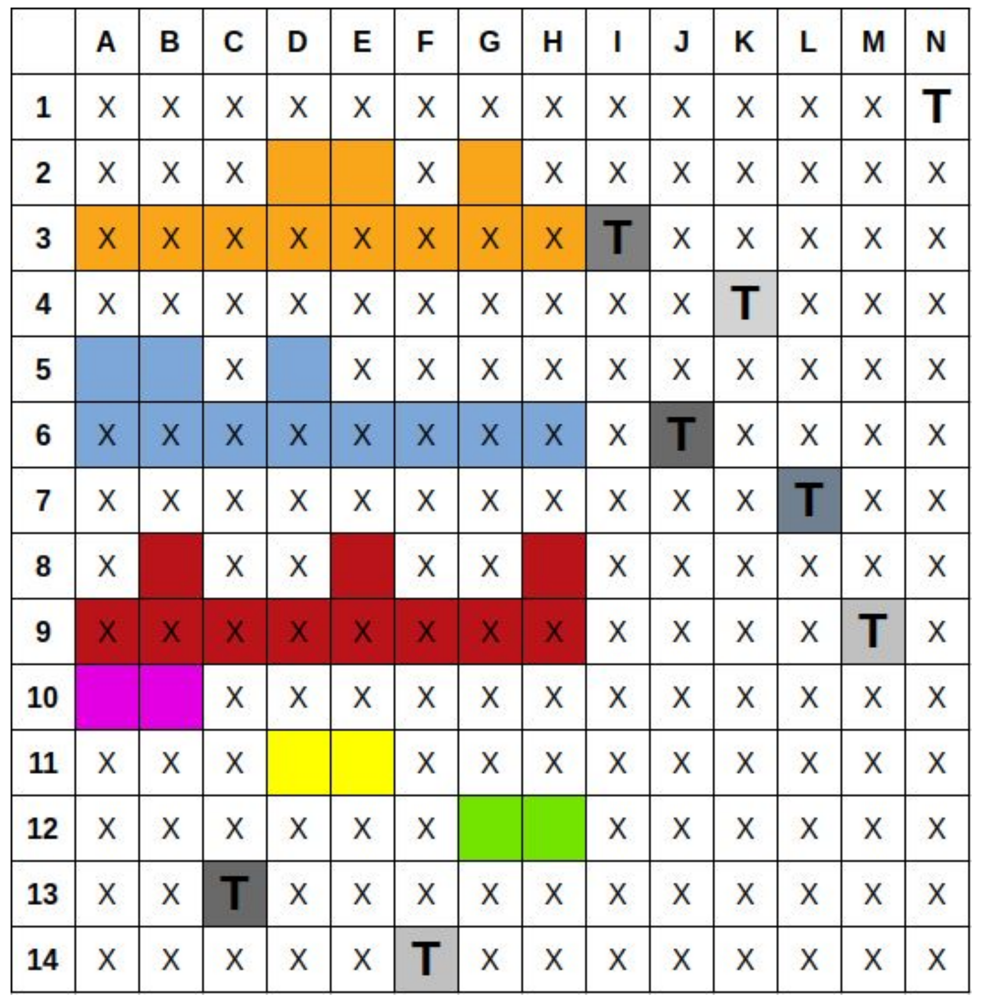
The IFF and OR gadgets

The IFF gadget consists of two sub-gadgets connected such that each of the sub-gadgets has its variable parks (the purple parks) set to the inverse of the other gadget. The gadget has two configurations corresponding to the setter park {A1,A2} being set to True and False respectively, and can be extended to include an arbitrary number of variable parks.



The IFF Gadget

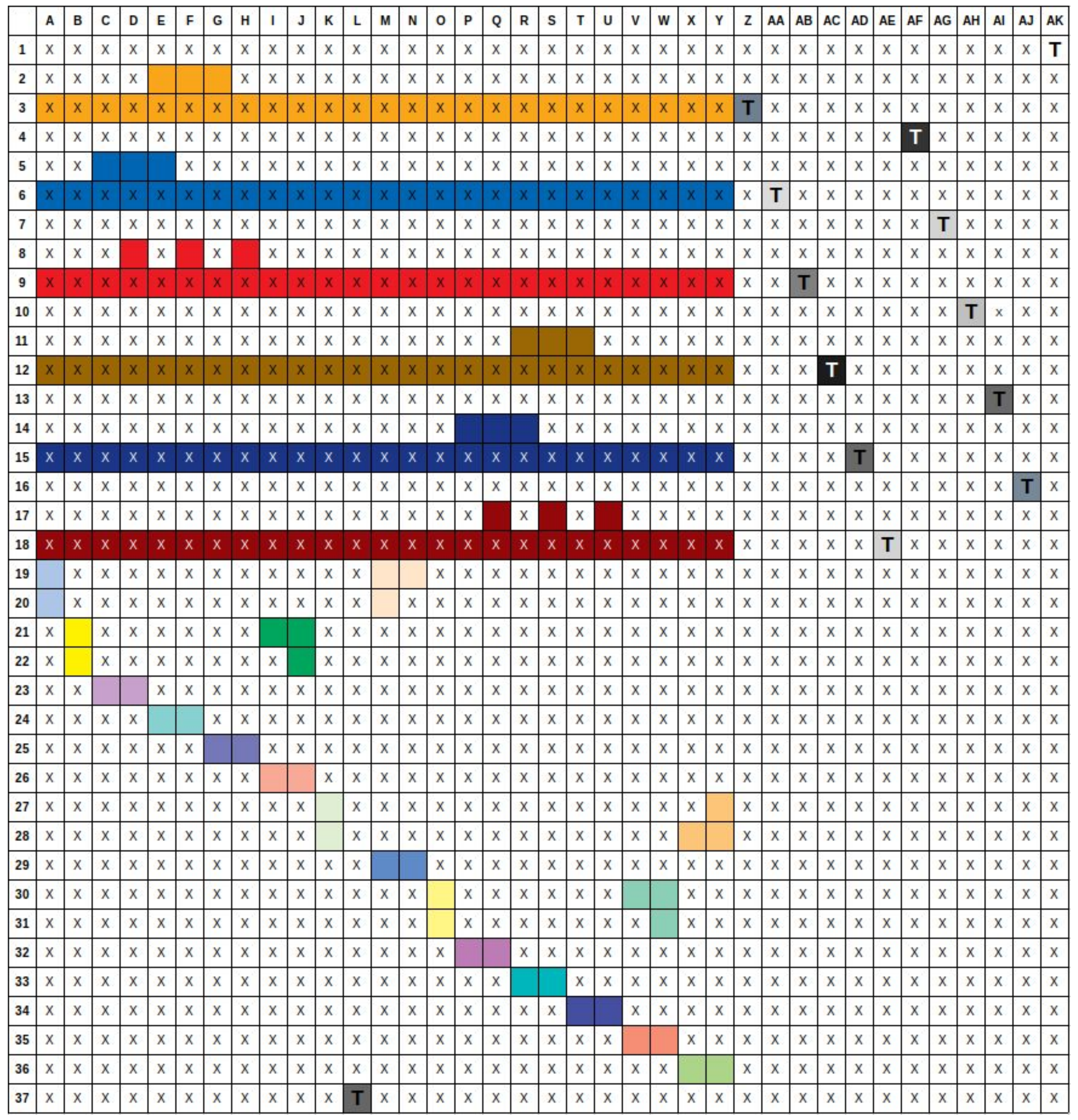
The OR gadget is the equivalent of a ternary OR expression with the purple, yellow and green parks as variable parks. The only inconsistent configuration arises when all three variable parks are set to false, and each of the remaining 7 possible variable assignments corresponds to a unique consistent state of gadget.



The OR Gadget

Example

The following is a simple example that depicts how the gadgets are used in conjunction represent any statement in 3CNF form. Figure 3 depicts the Parks equivalent of $(X \vee X \vee X) \wedge (!X \vee !X \vee !X)$ which uses two ternary OR gadgets and a IFF-NOT gadget. The value of X may be set to true by placing a tree in *B21*, and to false by placing a tree in *B20*. It is quite straightforward to verify that there are no solutions to the puzzle, which is consistent with the fact that the original expression is not satisfiable.



The Parks equivalent of $(X \vee X \vee X) \wedge (!X \vee !X \vee !X)$

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References

"This poster is based on work described in "Parks Puzzle is NP-Complete", K. Aditya Karan, available at https://portfolios.cs.earlham.edu/index.php/author/akamir16/